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ABSTRACT

This study sought to determine the basis for young children's understanding of fundamental addition and subtraction processes, and to expose any limitations on such arithmetic reasoning. Thirty-six two-year-olds and 36 three-year-olds participated in six experiments which examined children's relational quantity judgments about pairs of arrays in static and transformation conditions. There were two principal conclusions drawn from the investigation. First, the judgments of two-year-olds reflected chance responses on all problems, whereas three-year-olds' judgments indicated an understanding of the fundamental addition and subtraction processes studied. This difference in performance attests to a qualitative difference in the arithmetic knowledge of these two age groups. Second, although three-year-olds manifested an understanding of fundamental addition and subtraction, approximately half of these children based their reasoning exclusively on quantity transformation. Their failure to evaluate the outcome of the quantity transformation in making relational judgments about altered arrays reveals an important limitation on early arithmetic reasoning. (Author/MK).

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Young Children's Understanding of Addition and Subtraction: A Contextual Analysis

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Recent research on the development of number concepts indicates that preschool children are able to make accurate judgments about sets of objects which have undergone addition or subtraction (Beilin, 1968; Brush, 1978; Gelman, 1972; Siegler, in press). The converging evidence from these investigations supports the conclusion that children as young as three years of age exhibit a level of arithmetic knowledge which can be characterized as "fundamental" addition and subtraction. They understand that addition and subtraction are quantity-changing transformations and the direction of the quantity change is specific to the transformation. That is, young children's performance across a variety of addition and subtraction tasks reflects the knowledge that adding to a set increases its quantity and subtracting from a set decreases its quantity. The issue, however, of what logical abilities underlie this level of arithmetic knowledge has not been clearly resolved (Gelman & Gallistel, 1978; Cooper et al., Note 1).

The present study sought to determine the basis for young children's understanding of fundamental addition and subtraction, and to expose any limitations on such arithmetic reasoning. More specifically, the research addressed three questions. First, are there developmental differences in children's performance on addition and subtraction problems varying in quantitative context (i.e., initial equality or inequality of the sets of objects in the problem)? Second, when children solve an addition or subtraction problem, on what component(s) of the problem do they base their judgments? For example, in the case of the problems in the present study which entailed comparison sets of objects, do children reason exclusively on the quantity transformation performed on one set or do they also evaluate the outcome of that transformation on the comparison sets? Third, when children do not solve certain addition and subtraction problems,

do they use any systematic strategies which would produce these errors?

The results are derived from six experiments examining children's relational quantity judgments about pairs of arrays in static and transformation conditions. Thirty-six 2-year-olds (18 females and 18 males) between 2.5 and 3.0 years of age (mean = 2 years, 9 months) and 36 3-year-olds (18 females and 18 males) between 3.0 and 4.0 years of age (mean = 3 years, 6 months) participated in the six experiments. The pairs of arrays in each experiment varied along specific numerical and spatial dimensions. For the purposes of this report, the principal distinction among the experiments was that experiments 1-3 employed pairs of arrays that were initially equal, and experiments 4-6 employed pairs of arrays that were initially unequal. The manipulation of initially equal and unequal arrays provided variation in quantitative context for the addition and subtraction problems.

All the experiments consisted of two phases - a static array condition and an altered array condition. In the initial phase, children were presented with pairs of linear arrays of poker chips that ranged in set size from 4 to 7. They were requested to make relational judgments of more, less and same with respect to these static arrays. Children then observed a transformation of one of the arrays (addition of two chips, subtraction of two chips, or spatial relocation). In the second phase, relational judgments about the altered arrays were again obtained. The accuracy of children's relational judgments was analyzed in a 2(Age: 2 vs. 3-year-olds) x 2(Array state: static vs. altered) x 3(Type of relational judgment: more vs. less vs. same) repeated-measures analysis of variance for each experiment. One intriguing result was a significant effect for Array state which varied with the initial quantitative context of the experiment. That is, relational judgments were more accurate on transformed arrays than

static arrays in the initial equality experiments, but not in the initial inequality experiments. This finding prompted a more detailed examination of children's understanding of quantity transformations.

The first question in this analysis concerned whether there are developmental differences in children's ability to solve addition and subtraction tasks that varied in quantitative context. It was assumed that knowledge of fundamental addition and subtraction would be sufficient for success on selected addition and subtraction problems, designated "numerical agreement" problems, in which the outcome of the transformation was consistent with the particular quantity transformation. In problems of this type, addition to one array produced a greater quantity than the comparison array, and subtraction from one array produced a lesser quantity than the comparison array. This assumption led to two predictions: (1) if young children understand fundamental addition and subtraction, then they will make accurate judgments following a quantity transformation regardless of the quantitiative context of the problem; and (2) if in fact an understanding of fundamental addition and subtraction develops around three years of age, then 3-year-olds will make more accurate judgments following a quantity transformation than 2-year-olds.

Children's more and less judgments on numerical agreement problems were scored as correct or incorrect in the static condition of the arrays and after an addition/subtraction transformation. Tables 1 and 2 summarize the number of children in each experiment who made correct and incorrect judgments. A chi-square analysis of these data simply replicated the analysis-of-variance results. That is, children at both ages performed significantly better ($p < .01$ on most problems) following quantity transformations on equal arrays (experiments 1-3), but not following quantity

quantity transformations on unequal arrays (experiments 4-6). The distribution of children's performance patterns within the contingency tables provided the key to unraveling these findings.

As revealed in Tables 1 and 2, 2-year-olds' judgments on the altered arrays were distributed about equally between correct and incorrect responses. In contrast, 3-year-olds' judgments were predominantly correct. This developmental difference in children's performance following a quantity transformation was observed in all six experiments. It is apparent that the quantitative context effect exposed by the chi-square tests and the analyses of variance was due to the uniformly poor performance of both age groups on static equality arrays. In summary, the performance patterns of 2-year-olds suggested chance responding on addition and subtraction problems, whereas the patterns of 3-year-olds were consistent with an understanding of fundamental addition and subtraction. The data thus confirmed both predictions of this analysis.

The results of the first analysis indicated that 3-year-old children were able to solve addition and subtraction problems in which the outcome of the transformation was in numerical agreement with the quantity transformation. The second question addressed the basis for these accurate judgments. The purpose was to determine whether young children's arithmetic reasoning was informed by the quantity transformation performed on one array alone, or by the outcome of the quantity transformation on the comparison between arrays. In order to distinguish between these two bases for performance, children's judgments on numerical agreement problems were compared with their judgments on numerical non-agreement problems in which the outcome of the transformation was not consistent with the particular quantity transformation. That is, addition to one array produced a lesser quantity than the comparison array, and subtraction from

one array produced a greater quantity than the comparison array.

The second analysis examined two hypotheses. If young children consider only the quantity transformation in reasoning about addition and subtraction, then judgments will be accurate on numerical agreement problems but not on numerical non-agreement problems. On the other hand, if young children evaluate the outcome of the quantity transformation in reasoning about addition and subtraction, then no difference in the accuracy of judgments on numerical agreement and numerical non-agreement problems will be observed.

Children's more and less judgments on pairs of numerical agreement and numerical non-agreement problems were scored as correct or incorrect in the altered condition of the arrays. Tables 3 and 4 summarize the number of children in experiments 4, 5 and 6 who made correct and incorrect judgments. An inspection of children's performance patterns within the contingency tables revealed two interesting findings.

First, there was a marked developmental trend in children's understanding of addition and subtraction which supported the conclusion of the first analysis. The judgments of 2-year-olds were randomly distributed between correct and incorrect responses on both types of problems, and they did not fit either of the two hypothesized patterns. However, 3-year-olds' judgments clearly corresponded to one of the two patterns. Second, 3-year-old children were distributed about equally between success and failure on numerical non-agreement problems even though they succeeded on numerical agreement problems. These results suggested that at least half of the 3-year-olds were reasoning about addition and subtraction on the basis of the quantity transformation alone. An error analysis confirmed this interpretation.

The final question dealt with the types of strategies that led

children to err on numerical non-agreement problems and to solve numerical agreement problems. An analysis revealed that 88% of the 3-year-olds' responses in this error pattern followed an "absolute" strategy. Specifically, they reasoned as though addition to an array always results in a greater quantity and, conversely, subtraction from an array always results in a lesser quantity. This finding provided further support for the argument that some 3-year-olds only considered the quantity transformation in their reasoning about addition and subtraction.

There are two principal conclusions to be drawn from the present investigation. First, the judgments of 2-year-olds reflected chance responding on all problems, whereas 3-year-olds' judgments indicated an understanding of fundamental addition and subtraction. This difference in performance attests to a qualitative difference in the arithmetic knowledge of these two age groups. Second, although 3-year-olds manifested an understanding of fundamental addition and subtraction, approximately half of these children based their reasoning exclusively on the quantity transformation. Their failure to evaluate the outcome of the quantity transformation in making relational judgments about altered arrays reveals an important limitation on early arithmetic reasoning.

Our evidence that young children possess knowledge of fundamental addition and subtraction is generally consistent with the results of previous investigations (e.g., Gelman & Gallistel, 1978). However, the interpretation offered for the evidence is different. We wish to argue that the abilities and limitations at this level of arithmetic knowledge reflect a pre-operational reasoning about number. In particular, the treatment of addition and subtraction operations as one-directional functions (Piaget et al., 1977) is sufficient to explain the 3-year-olds' understanding that adding enlarges a collection and subtracting diminishes a collection.

Reference Note

1. Cooper, R., Starkey, P., Blevins, B., Goth, P., & Leitner, E. Number development: Addition and subtraction. Unpublished manuscript, University of Texas, 1978.

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Table 1

RESPONSES OF MORE AND LESS TO STATIC AND ADDITION TRANSFORMATION CONDITIONS BY AGE (Number of Subjects)[a]

Age and quantity terms	Static	EXPERIMENT															
		1[b] Addition		2[c] Addition		3[b] Addition		3[c] Addition		4[b] Addition		5[c] Addition		6[b] Addition		6[c] Addition	
		-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+
2-year-olds																	
More	-	3	9	5	7	5	7	7	5	1	2	2	5	2	1	3	5
	+	0	0	0	0	0	0	0	0	5	4	3	2	3	6	3	1
Less	-	6	6	5	6	8	4	7	5	0	1	3	3	2	1	2	4
	+	0	0	1	0	0	0	0	0	4	7	3	3	2	7	5	1
3-year-olds																	
More	-	2	6	3	7	0	10	3	8	1	2	1	2	0	2	0	4
	+	0	4	0	2	1	1	1	0	1	8	0	9	1	9	0	8
Less	-	1	6	3	7	0	12	5	7	2	3	2	1	0	2	0	5
	+	1	4	1	1	0	0	0	0	2	5	0	9	1	9	0	7

Note. - N=12 subjects in each experiment per age group.
"+" = pass item and "-" = fail item under a given condition.

[a]This analysis included all experimental items in which the altered array was in numerical agreement with the addition transformation. Addition to one array yielded a quantity that was greater than the comparison array.

[b]Refers to experimental items in which the altered array was in spatial agreement with the addition transformation. Addition to one array yielded a quantity in a linear configuration that was longer than the comparison array.

[c]Refers to experimental items in which the altered array was in spatial non-agreement with the addition transformation. Addition to one array yielded a quantity in a linear configuration that was shorter than the comparison array.

Table 2

RESPONSES OF MORE AND LESS TO STATIC AND SUBTRACTION TRANSFORMATION CONDITIONS BY AGE (Number of Subjects)[a]

Age and quantity term	Static	EXPERIMENT													
		1[b] Subtraction		2[c] Subtraction		3[b] Subtraction		3[c] Subtraction		4[b] Subtraction		5[c] Subtraction		6[b] Subtraction		6[c] Subtraction	
		-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+
2-year olds																	
More	-	2	9	2	9	5	7	7	5	2	0	1	2	1	3	3	3
	+	0	1	1	0	0	0	0	0	2	8	5	4	0	8	2	4
Less	-	6	6	3	9	1	11	5	7	1	3	2	0	0	3	3	5
	+	0	0	0	0	0	0	0	0	2	6	4	6	0	9	1	3
3-year olds																	
More	-	2	8	4	7	1	9	2	9	1	2	1	4	1	2	0	4
	+	0	2	0	1	0	2	0	1	0	9	0	7	0	9	1	7
Less	-	2	8	4	8	1	10	2	9	1	3	1	4	2	1	0	4
	+	1	1	0	0	0	1	0	1	1	7	0	7	0	9	1	7

Note. - N=12 subjects in each experiment per age group.
"+" = pass item and "-" = fail item under a given condition.

[a]This analysis included all experimental items in which the altered array was in numerical agreement with the subtraction transformation. Subtraction from one array yielded a quantity that was less than the comparison array.

[b]Refers to experimental items in which the altered array was in spatial agreement with the subtraction transformation. Subtraction from one array yielded a quantity in a linear configuration that was shorter than the comparison array.

[c]Refers to experimental items in which the altered array was in spatial non-agreement with the subtraction transformation. Subtraction from one array yielded a quantity in a linear configuration that was longer than the comparison array.

Table 3

RESPONSES OF MORE AND LESS TO NUMERICAL AGREEMENT AND NUMERICAL NON-AGREEMENT WITH ADDITION TRANSFORMATION BY AGE

(Number of Subjects)

Age and quantity term	Agreement	EXPERIMENT 4[a] Non-Agreement -	4[a] +	5[b] Non-Agreement -	5[b] +	6[a] Non-Agreement -	6[a] +	6[b] Non-Agreement -	6[b] +
2-year olds									
More	-	4	2	2	3	2	3	5	1
	+	3	3	3	4	3	4	2	4
Less	-	3	1	2	4	2	2	3	4
	+	5	3	1	5	3	5	1	4
3-year olds									
More	-	1	1	1	0	0	1	0	0
	+	6	4	4	7	5	6	3	9
Less	-	2	2	2	0	0	1	0	0
	+	6	2	5	5	5	6	5	7

Note. - N=12 subjects in each experiment per age group.
"+" = pass item and "-" = fail item under a given condition.

[a] Refers to experimental items in which the altered array was in spatial agreement with the addition transformation. Addition to one array yielded a quantity in a linear configuration that was longer than the comparison array.

[b] Refers to experimental items in which the altered array was in spatial non-agreement with the addition transformation. Addition to one array yielded a quantity in a linear configuration that was shorter than the comparison array.

Table 4

RESPONSES OF MORE AND LESS TO NUMERICAL AGREEMENT AND NUMERICAL NON-AGREEMENT WITH SUBTRACTION TRANSFORMATION BY AGE

(Number of Subjects)

Age and quantity term	Agreement	EXPERIMENT 4[a] Non-Agreement -	4[a] Non-Agreement +	5[b] Non-Agreement -	5[b] Non-Agreement +	6[a] Non-Agreement -	6[a] Non-Agreement +	6[b] Non-Agreement -	6[b] Non-Agreement +
2-year olds									
More	-	0	4	3	3	1	0	2	3
	+	7	1	4	2	3	8	2	5
Less	-	2	1	4	2	0	0	0	4
	+	7	2	2	4	8	4	5	3
3-year olds									
More	-	0	1	1	0	1	0	0	1
	+	6	5	4	7	6	5	3	8
Less	-	1	1	1	0	1	1	0	1
	+	5	5	4	7	6	4	3	8

Note. - N=12 subjects in each experiment per age group.
"+" = pass item and "-" = fail item under a given condition.

[a] Refers to experimental items in which the altered array was in spatial agreement with the subtraction transformation. Subtraction from one array yielded a quantity in a linear configuration that was shorter than the comparison array.

[b] Refers to experimental items in which the altered array was in spatial non-agreement with the subtraction transformation. Subtraction from one array yielded a quantity in a linear configuration that was longer than the comparison array.